Pricing electricity forward contracts under observable information in JEPX (Japan Electric Power Exchange)

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Japan Electric Power Exchange: JEPX

- Match sell and buy orders for delivering a **fixed amount of electricity** (kWh) in **nine areas**, Hokkaido, Tohoku, Tokyo, Chubu, Hokuriku, Kansai, Chugoku, Shikoku, and Kyushu
- From short term (**half hour**) to long term (**1 week—1 year**)

Spot contract

Forward contract

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Japan Electric Power Exchange: JEPX

- **Spot electricity** in JEPX: Every half hour, 48 products a day. Orders are closed at 9am 1 day before the delivery.
- Unique spot price in the **entire 9 areas of Japan** ➔ System price
- **Forward contracts**: Written on the system price
  
  **Illiquid, only traded several times a year!**
Outline

Forward prices under observable information (JEPX spot price)

1. Forward contracts in JEPX, its expression using daily forward prices, and an application of Esscher transform

2. Derivation of daily forward price via Esscher transformation
   - Modeling of underlying spot price using state space equations
   - Comparison with Girsanov transformation

3. Empirical simulation

4. Concluding remarks

Forward contracts in JEPX

- Financial settlement between forward and system prices for the delivery period from 1 week to 1 year in the future.

\( S_t : \) Average system price at day \( t \) (spot price)

\( F_{t,T}^{(m)} : \) Forward price with \( m \) days delivery starting at day \( T \) (> \( t \))

\[
	ext{Total payoff: } \sum_{u=T}^{T+m-1} [S_u - F_{t,T}^{(m)}] = \sum_{u=T}^{T+m-1} S_u - m \times F_{t,T}^{(m)}
\]
Expression using daily forward prices

\[ F_{t,T} := F_{t,T}^{(1)} : \text{Daily forward price of delivering at day } T \text{ only} \]

Total payoff:

\[
\sum_{u=T}^{T+m-1} [S_u - F_{t,u}] = \sum_{u=T}^{T+m-1} S_u - \sum_{u=T}^{T+m-1} F_{t,u} = \sum_{u=T}^{T+m-1} S_u - m \times F_{t,T}^{(m)}
\]

No arbitrage!

Float CF (Stochastic)

Float CF (Deterministic)

Spot and daily forward prices

\[ F_{t,T} := F_{t,T}^{(1)} : \text{Daily forward price of delivering at day } T \text{ only} \]

Payoff: \( S_T - F_{t,T} \rightleftharpoons \text{Function of } S_T \)

- Connection between spot and forward prices

\( S_t : \text{Spot price process on } (\Omega, \mathcal{F}, \mathbb{P}, \{\mathcal{F}_t\}) \)

\[
\ln S_t = f(t) + \eta_t \iff S_t = \exp[f(t) + \eta_t]
\]

- \( f(t) : \text{Deterministic function of } t \)
  - Seasonal and long term trends + Day-of-the-week effects

- \( \eta(t) : \mathcal{F}_t\)-adopted stochastic process (observed at time \( t \))
Forward price via Esscher transform (Kijima and Tanaka‘07)

- **Esscher transform**: \( S_T = \exp[f(T) + \eta_T] \)
  - change of measure:
  \[
  Z_T := \frac{e^{\lambda \eta_T}}{\mathbb{E}[e^{\lambda \eta_T}]}, \quad \lambda \in \mathbb{R}: \text{Esscher parameter}
  \]
  \[
  \mathbb{P}(A) := \mathbb{E} [I_A Z_T], \quad A \in \mathcal{F}_T
  \]

- **Forward price**:
  \[
  F_{t,T} = \mathbb{E} [S_T | \mathcal{F}_t] = \frac{1}{Z_t} \mathbb{E} [S_T Z_T | \mathcal{F}_t] = \frac{1}{Z_t} \mathbb{E} [e^{f(T) + \eta_T Z_T} | \mathcal{F}_t]
  \]
  \[
  Z_t := \mathbb{E} \left[ \frac{e^{\lambda \eta_T}}{\mathbb{E}[e^{\lambda \eta_T}]} \right] \mathcal{F}_t
  \]

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Forward price and risk premium

- Positive (negative) \( \lambda \) reflects on the sellers’ (buyers’) risk premium.
- \( \lambda = 0 \) (i.e., \( Z_T = 1 \)) is risk neutral so that \( F_{t,T} = \mathbb{E} [S_T | \mathcal{F}_t] \).

<table>
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<tr>
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</thead>
<tbody>
<tr>
<td>( \lambda )</td>
<td>( \longrightarrow )</td>
<td>( \longleftarrow )</td>
<td>( \lambda = 0 )</td>
</tr>
<tr>
<td>Price</td>
<td>( \longrightarrow )</td>
<td>( \longleftarrow )</td>
<td>Risk neutral</td>
</tr>
</tbody>
</table>

\( \lambda \) may be implied by matching with the realized forward price.

Forward price with \( m \) days delivery:
\[
F_{t,t+\tau}^{(m)} = \frac{1}{m} \sum_{u=T}^{T+m-1} F_{t,u}
\]
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Modeling of underlying spot price

\[ \ln S_t = f(t) + \eta_t \iff S_t = \exp[f(t) + \eta_t] \]

- \( f(t) \): Seasonal and long term trends + Day-of-the-week effects
- \( \eta(t) \): \( F_t \)-adopted stochastic process (observed at time \( t \))

Observed equation: \( \eta_t = C^T x_t \), State equation: \( x_t = Ax_{t-1} + Bw_t \)

\( x_t \in \mathbb{R}^n \): state variable, \( w_t \in \mathbb{R}^m \): white noise
\( A, B, C \in \mathbb{R}^{n \times n}, \mathbb{R}^{n \times m}, \mathbb{R}^n \): constant matrices

- \( \eta_t \sim ARMA(p,q) \) e.g., Durbin & Koopman ‘01
- Multi-factor model e.g., Gibson & Schwartz ‘90, Schwartz & Smith ‘00
Computation of forward price

\[ S_T = \exp[f(T) + \eta_T] \]
\[ x_T = Ax_{T-1} + Bw_T = A^{T-t}x_t + \sum_{s \geq 1} A^{T-t-s}Bw_{t+s} \]
\[ \eta_T = C^T x_T \]

- \[ g_{t,T} := \mathbb{E}[\eta_T | \mathcal{F}_t] \]: Predicted value of \( \eta_T \).
- \( \eta_T - g_{t,T} \): Prediction error.

\[ g_{t,T} = \mathbb{E}[C^T x_T | \mathcal{F}_t] = C^T \mathbb{E}[x_T | \mathcal{F}_t] = C^T A^{T-t}x_t \]

\[ \eta_T - g_{t,T} = C^T \sum_{s \geq 1} A^{T-t-s}Bw_{t+s} \rightarrow \text{Conditionally independent of } \mathcal{F}_t \]

\[ F_{T,T} = \frac{1}{Z_t} \mathbb{E}[e^{f(T)+\eta_T Z_T} | \mathcal{F}_t] = e^{f(T)+g_{t,T}} \frac{\mathbb{E}[e^{(\lambda+1)(\eta_T - g_{t,T})}]}{\mathbb{E}[e^{\lambda(\eta_T - g_{t,T})}]} \]

\[ = \exp\left(f(T) + g_{t,T} + \phi_{\eta_T - g_{t,T}} (\lambda + 1) - \phi_{\eta_T - g_{t,T}} (\lambda)\right) \]

\( \phi_{\eta_T - g_{t,T}} \): Cumulant generating function

Comparison with Girsanov transformation

- **Continuous time model:**
  \[ dx_t = (Ax_t + D)dt + BdW_t, \quad \eta_t = C^T x_t \]
  \( W_t \in \mathbb{R}^n \): \( n \)-dimensional Brownian motion

  - **Girsanov:**
    \[ Z_T = \frac{e^{\int_0^T \theta_t dw_t}}{\mathbb{E}[e^{\int_0^T \theta_t dw_t}]}, \quad \theta_t \in \mathbb{R}^n \]: Market price of risk
    \[ dW_t \text{ under } \mathbb{P} \to d\tilde{W}_t + \theta_t dt \text{ under } \tilde{\mathbb{P}} \]

  - **Esscher:**
    \[ Z_T = \frac{e^{\lambda \eta_T}}{\mathbb{E}[e^{\lambda \eta_T}]}, \quad \lambda \in \mathbb{R} \]: Esscher parameter
    \[ \eta_T \text{ under } \mathbb{P} \to \tilde{\eta}_T + \lambda \sigma_{\tilde{\eta}_T}^2 \text{ under } \tilde{\mathbb{P}} \]

Equivalence condition:
\[ \theta_t = \lambda C^T e^{A(T-t)} B \]
Comparison with Girsanov transformation

Proof: \( dx_t = (Ax_t + D)dt + BdW_t, \quad \eta_t = C^T x_t \)

\[
x_T = e^{AT}x_0 + e^{AT} \int_0^T e^{-At}BDdt + e^{AT} \int_0^T e^{-At}BdW_t
\]

\[
\eta_T = C^T x_T = C^T e^{AT} x_0 + \int_0^T C^T e^{A(T-t)} Ddt + \int_0^T C^T e^{A(T-t)} BdW_t
\]

\[
Z_T = \frac{e^{\lambda \eta_T}}{\mathbb{E}[e^{\lambda \eta_T}]} = \frac{e^{\lambda \int_0^T C^T e^{A(T-t)} BdW_t}}{\mathbb{E}[e^{\lambda \int_0^T C^T e^{A(T-t)} BdW_t}]} = \frac{e^{\int_0^T \theta_t dW_t}}{\mathbb{E}[e^{\int_0^T \theta_t dW_t}]}
\]

Esccher transform

Girsanov transform

Equivalence condition: \( \theta_t = \lambda C^T e^{A(T-t)} B \)

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Empirical simulation

- Data period: 2012/4/1–2016/12/31 (N = 1736)
  - Spot price (daily average of 48 system prices released every day)

- Daily forward price (Esscher):
  \[ F_{t,T} = e^{g_{t,T}} \frac{\mathbb{E}[e^{(\lambda+1)(\eta_T-g_{t,T})}]}{\mathbb{E}[e^{\lambda(\eta_T-g_{t,T})}]} \]

- \( g_{t,T} = \mathbb{E}[\eta_T|F_t] \): Predicted value of \( \eta_T \).
- \( \eta_T - g_{t,T} \): Prediction error.

Step 1) Estimate trend function \( f(t) \) and residual \( \eta_t \) using underlying spot price data \( S_t, t = 1, ..., N \).

Step 2) Apply \( AR(q) \) for \( \{\eta_t\} \) and compute distribution of prediction errors \( \eta_T - g_{t,T} \).

Step 3) Comparison with realized forward prices.

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Step 1) Estimation of seasonal trend using GAM

- Generalized Additive Model (GAM; Hastie and Tibshirani‘90)

\[
\ln S_t = h(Seasonal_t) + \alpha \times t + \sum_{j=0}^{6} \beta_j \times Dummy_{i} + c + \eta_t
\]

- \( f(t) \)
- \( Dummy_0 \): Holiday dummy, \( Dummy_{1-6} \): Day dummy (0 or 1)
- \( Seasonal_t \): Seasonal trend dummy (e.g., 1,…,365 (366))

- Yamada et al. ’14: Apply GAM with repetitive data with

\[
Seasonal_t = 1, ..., 365 (366)  \\
Seasonal_t = -364 (-365), ..., 0,  \\
Seasonal_t = 366 (367), ..., 730 (731).
\]

Choose spline function for \( Seasonal_t = 1, ..., 365 (366) \).
Estimation of trend function by GAMs

- Data period: 2012/4/1–2016/12/31 (N = 1736)

\[ \ln S_t = h(Seasonal_t) + \alpha \times t + \sum_{j=0}^{6} \beta_j \times Dummy_i + c + \eta_t \]

\[ \eta_t \sim N(0, \sigma^2) \]

- \( \beta_j \) (Mon—Sat and Holiday)

Step 2) Estimation of prediction errors

- Predicted values for \( \{\eta_t\} \) using AR(q) model with \( q = q_{BIC} \):

  \[ k: \text{Current date}, \ L: \text{Learning data period}, \ \tau: \text{prediction horizon} \]

Given learning data, construct \( AR(q_{BIC}) \). Predict \( \eta_{k+\tau} \) by \( \hat{\eta}_{k+\tau} = \mathbb{E}[\eta_{k+\tau} \mid F_k] \) and compute \( \hat{\eta}_{k+\tau} - \hat{\eta}_{k+\tau} \) for \( k = L, \ldots, N - \tau \).

- Daily forward price using empirical distribution:

\[ \hat{F}_{t, t+\tau} = e^{\left(f(t+\tau) + \hat{\eta}_{k+\tau}\right)} \times \frac{\sum_{k=L}^{N-\tau} e^{(\lambda+1)\left(\eta_{k+\tau} - \hat{\eta}_{k+\tau}\right)}}{\sum_{k=L}^{N-\tau} e^{\lambda\left(\eta_{k+\tau} - \hat{\eta}_{k+\tau}\right)}}, \ t = L, \ldots, N - \tau \]
Term structure of prediction errors

- Prediction errors of $AR(q_{BIC})$ when $L = 90$ days:

\[ MAE = \frac{1}{N - \tau - L + 1} \sum_{k=L}^{N-\tau} |\eta_{k+\tau} - \hat{\eta}_{k+\tau}|, \quad SD = \sqrt{\frac{1}{N - \tau - L} \sum_{k=L}^{N-\tau} \left( (\eta_{k+\tau} - \hat{\eta}_{k+\tau} - \bar{\eta}_{k+\tau}) \right)^2} \]

Empirical estimation of daily forward prices

- Estimated value of daily forward price:

\[ \hat{F}_{t,t+\tau} = e^{f(t+\tau)} + \hat{g}_{k,k+\tau} \times \frac{\sum_{k=L}^{N-\tau} e^{(1+1)(\eta_{k+\tau} - \hat{\eta}_{k+\tau})}}{\sum_{k=L}^{N-\tau} e^{(1+1)(\eta_{k+\tau} - \hat{\eta}_{k+\tau})}}, \quad t = L, ..., N - \tau \]

- MAE between the daily forward price with $\lambda = 0$ and spot price

- Risk premium rate $\hat{RPR}_{\lambda} = \frac{\hat{F}_{t,t+\tau}}{\hat{F}_{t,t+\tau}} |_{\lambda = 0}$
Step 3) Comparison with realized forward prices

- **Realized forward prices in JEPX (2012/4/1–2016/12/31):**

<table>
<thead>
<tr>
<th>Year of transaction</th>
<th>2012</th>
<th>2013</th>
<th>2014</th>
<th>2015</th>
<th>2016</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 week delivery</td>
<td>0</td>
<td>0</td>
<td>8</td>
<td>2</td>
<td>(1)</td>
</tr>
<tr>
<td>1 month delivery</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>3</td>
<td>-</td>
</tr>
</tbody>
</table>

11 forward prices

- **Estimated forward price:**

\[
\hat{F}_{t,t+\tau}^{(m)} = \frac{1}{m} \sum_{u=t+\tau}^{t+\tau+m-1} \hat{F}_{t,u}
\]

\(\lambda\) may be implied by matching with the realized forward price.

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<td>Price</td>
<td>(\rightarrow)</td>
<td>(\rightarrow)</td>
<td>Risk neutral</td>
</tr>
</tbody>
</table>

- Risk premium of realized forward price?
- Estimated sample path of forward price with implied \(\lambda\)?

Step 3) Comparison with realized forward prices

- **Forward payoff:**

\[
\sum_{u=t+\tau}^{t+\tau+m-1} [S_u - F_{t,T}^{(m)}] = \sum_{u=t+\tau}^{t+\tau+m-1} [S_u - F_{t,u}] = m \left( \frac{1}{m} \sum_{u=t+\tau}^{t+\tau+m-1} S_u - \frac{1}{m} \sum_{u=t+\tau}^{t+\tau+m-1} F_{t,u} \right)
\]

Payoff normalized by number of days

- Predicted vs. realized values of the underlying vs. forward payoffs?
Predicted and realized values vs. forward payoff

“Realized – Predicted of the underlying” & Normalized forward payoff

- Realized payoff is significantly negative.

- Implied $\lambda$ by AR prediction is always positive.

---

**Case with maximum payoff size**

**AR prediction**

**RW prediction**

- In the case of RW prediction, the forward price fluctuates with the daily spot price and is affected by “spikes.”
Case with minimum payoff size

- In the case of **AR prediction**, the forward price is smooth and seems to reflect on the **long term average** of spot prices.

Concluding remarks

- **Modeling of spot price dynamics** and **forward prices**
  - Trend function ➔ Generalized additive model (GAM)
  - Stochastic part ➔ State space equation formula
  - Forward price ➔ Esscher transformation

- **Empirical simulation**
  - In the case of **RW prediction**, the **forward price** fluctuates with the **daily spot price** and is affected by “spikes.”
  - In the case of **AR prediction**, the forward price is smooth and seems to reflect on the **long term average** of spot prices.